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Branes on generalized calibrated submanifolds

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Abstract: We extend previous results on generalized calibrations to describe supersymmetric branes in supergravity backgrounds with diverse fields turned on, and provide several new classes of examples. As an important application, we show that supersymmetric D-branes in compactifications with field strength fluxes, and on SU(3)-structure spaces, wrap generalized calibrated submanifolds, defined by simple conditions in terms of the underlying globally defined, but non-closed, 2- and 3-forms. We provide examples where the geometric moduli of D-branes (for instance D7-branes in 3-form flux configurations) are lifted by the generalized calibration condition. In addition, we describe supersymmetric D6-branes on generalized calibrated 3-submanifolds of half-flat manifolds, which provide the mirror of B-type D-branes in IIB CY compactifications with 3-form fluxes. Supersymmetric sets of such D-branes carrying no homology charges are mirror to supersymmetric sets of D-branes which are homologically non-trivial, but trivial in K-theory. As an additional application, we describe models with chiral gauge sectors, realized in terms of generalized calibrated brane box configurations of NS- and D5-branes, which are supersymmetric but carry no charges, so that no orientifold planes are required in the compactification.

Keywords: D-branes, Supersymmetry and Duality.

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1. Introduction

Recently it is becoming manifest that the dynamics of string theory is particularly rich and interesting in the presence of backgrounds with non-trivial field strength fluxes (or their dual versions). One example of such situations is provided by the gauge/gravity correspondence [1], where the additional ingredients break conformal invariance and (partially) supersymmetry. Another situation of this kind is in compactifications of string/M - theory with field strength fluxes for p-form fields (see e.g. [2]–[6]), or in supersymmetric compactifications on non-Calabi-Yau manifolds of SU(3) structure (see e.g. [7]–[9]).

These compactifications show interesting properties, like moduli stabilization, warped geometries, and tractable supersymmetry breaking. Thus they represent an important step in constructing phenomenologically appealing string vacua (see [10, 11, 12] for explicit model building with semirealistic gauge sectors), and in understanding string vacua with generically few moduli.

An important question in this framework is the behaviour of D-branes in such backgrounds. There are several levels at which this question may be addressed. For instance, at the topological level, there are additional consistency conditions for the possibilities of brane wrapping in the presence of fluxes [13, 14]. Or, from the world-volume perspective, the fluxes may induce new terms in the action for world-volume fields (see e.g. [15, 16, 17]). In the present paper, we center on the similarly important aspect of characterizing supersymmetry preserving branes in such backgrounds.

In supersymmetric cases, these backgrounds are generalizations of flat space configurations or of compactification on special holonomy manifolds. In these simpler cases, supersymmetric branes are associated to calibrated submanifolds, which are volume minimizing. Thus, we may expect that supersymmetric branes in generalized supersymmetric backgrounds are associated to generalized calibrations. These have been studied in particular simple cases in [18, 19] and more extensively discussed in [20], see also [21]. They can be described as closed forms which, once restricted to the brane world-volume, provide not just the volume of the wrapped submanifold, but also contributions from additional coupling of the branes to the background. Alternatively, they contain a non-closed piece reproducing the volume of the wrapped submanifold, and which can be complemented by additional terms to a closed calibration. Hence branes on generalized calibrated submanifolds minimize not their volume, but rather their action.

In the present paper we describe D-branes in generalized calibrated submanifolds in supersymmetric backgrounds including non-trivial profiles for the metric, dilaton, and diverse p-form fields. Our analysis and examples extend previous results on generalized calibrations in [18]–[21] to cases with additional backgrounds (and also those obtained in [22] from the Dp-brane worldvolume perspective).

We moreover argue that the description in terms of generalized calibrations allows to characterize supersymmetric wrapped branes in compactifications with field strength fluxes and on non-Calabi-Yau SU(3)-structure manifolds.[1] We show that generalized calibrated branes can also be characterized by imposing conditions of the familiar kind on the natural

[1]The interplay between G-structures and generalized calibrations has been discussed in [20]

bispinor forms, namely the 3-form Ω, and a complexified version of the 2-form J, which are globally defined, but not closed. Thus, generalized calibrations provide the natural setup in which to describe supersymmetric branes in flux compactifications.

Finally, we exploit a further property of branes on generalized calibrated submanifolds, namely that they are stable and supersymmetric even though they may not carry any topological charge. This allows new possibilities for model building, by using compactifications with branes but no orientifold planes. We illustrate this possibility in a compactification including brane-box configurations of NS5-branes and D5-branes on generalized calibrated submanifolds, leading to a chiral gauge sector.

The paper is organized as follows. In section 2 we review calibrations and their relation to supersymmetry, and derive generalized calibrations from supersymmetry algebra considerations. In section 3 we present examples of D-branes on generalized calibrated submanifolds in backgrounds involving non-trivial metrics, dilaton profiles, and field strength fluxes. These include and extend previous examples in the literature.

In section 4 we introduce a warm-up example, and describe D3-branes in 3-form flux backgrounds, and a T-dual version of D4-branes in non-Calabi-Yau metric backgrounds. We show they correspond to generalized calibrations, and that this is crucial for them to be supersymmetric, since they turn out to carry no ($\mathbb{Z}$-valued) charge. In section 5 we propose a description of generalized calibrated submanifolds in compactifications with fluxes, or on SU(3)-structure manifolds, in terms of simple conditions on the globally defined 2- and 3-forms. We provide explicit examples of branes in 3-form flux backgrounds, and on their mirror half-flat manifolds, supporting the proposal. In addition we show that in certain cases the condition to be generalized calibrated fixes some of the moduli of the wrapped submanifolds. For D7-branes, this encodes the fact that supersymmetric fluxes make D7-brane geometric moduli massive, a fact of relevance in the proposal in [23] to stabilize Kähler moduli via gauge non-perturbative effects on D7-branes.

In section 6 we illustrate new model building possibilities with generalized calibrated branes, by describing a model with 4d chiral gauge sectors from generalized calibrated brane box configurations. Finally, section 7 contains some final remarks. We list some conventions in the appendix.

2. Calibrations and generalized calibrations

2.1 Standard calibrations

In this section, we work in a pure metric background with no other fields turned on. Let us start by reviewing the formal definition of calibration [24].[2]

Let M be a d-dimensional riemannian manifold and φ be a k-form on M, $k < d$. The form φ is said to be a calibration on M if: i) $d\varphi=0$ (φ is closed) and ii) for any tangent k-plane to M, ξ, it satisfies $\varphi|_\xi \leq vol_\xi$, where $\varphi|_\xi$ denotes the pullback of φ on ξ, and vol_ξ is the induced volume form on ξ. This pullback will typically be $\varphi|_\xi = \alpha \cdot vol_\xi$ for $\alpha \in \mathbb{R}$ and so $\varphi|_\xi \leq vol_\xi$ when $\alpha \leq 1$.

[2] For a nice recent review and broader bibliography, see [25] and references therein.

A k-dimensional submanifold of M, N, is said to be calibrated by φ if at any point $x \in N$, it satisfies $\varphi|_{T_x N} = vol_{T_x N}$, i.e. if condition (ii) of the calibration is saturated at each point of the submanifold. It is hence intuitively clear that such calibrated submanifold will be volume minimizing in its topological class, as stated by the theorem of Harvey and Lawson [24].

Calibrations are useful in string theory because they realize BPS-like conditions, and so allow to classify supersymmetric extended objects of the background. This can be derived from the supersymmetry algebra as follows. The presence of an extended object in a generic supergravity background leads to a deformation of the superalgebra, that should now include an appropriate central charge, see e.g. [26, 27]. Schematically, the anticommutator of supercharges in the presence of an extended p-dimensional object is[3]

$$\{Q_\alpha, Q_\beta\} = (\Gamma^M)_{\alpha\beta} P_M + (\Gamma_{M_1 \cdots M_p})_{\alpha\beta} Z^{M_1 \cdots M_p}\big|_\xi \,, \tag{2.1}$$

where ξ denotes the tangent plane to a generic point of the p-extended object's world-volume and $\Gamma_{M_1 \cdots M_p}$ is the antisymmetrized product of gamma matrices.

Contracting this expression with a Killing spinor of the background ϵ^α, we reach the well known condition

$$(Q\epsilon)^2 = K^M P_M + \Phi_{M_1 \cdots M_p} Z^{M_1 \cdots M_p}\big|_\xi \geq 0 \,, \tag{2.2}$$

where we have defined

$$\begin{aligned} K^M &= \bar{\epsilon}\, \Gamma^M \epsilon \\ Z^{M_1 \cdots M_p} &= dX^{M_1} \wedge \cdots \wedge dX^{M_p} \,. \end{aligned} \tag{2.3}$$

After this contraction the central extension becomes a p-form built up as a bispinor[4]

$$\Phi_{(p)} = \Phi_{M_1 \cdots M_p}\, dX^{M_1} \wedge \cdots \wedge dX^{M_p} \qquad \text{with} \qquad \Phi_{M_1 \cdots M_p} = \bar{\epsilon}\, \Gamma_{M_1 \cdots M_p}\, \epsilon \,. \tag{2.4}$$

General considerations imply that this central extension of the supertranslation algebra must be closed, as explicitly follows from the Killing spinor equations for ϵ. Its integral over the p-dimensional spatial brane volume provides the topological charge of the extended object. Therefore (2.2) is the well-known BPS condition for p-extended objects. In the rest frame of the object, where $P_\mu = (\mathcal{H}, 0, \ldots, 0)$ with $\mathcal{H}$ the hamiltonian (energy) density, the

[3] Our notation is as follows. We use M, N, ... for ambient (generally curved) spacetime indices, and $A, B, \ldots$ for tangent space indices. For the world-volume of extended objects, parametrized by coordinates denoted σ, we use indices μ, ν, ... for (curved) world-volume indices and a, b, ... for tangent space ones. Spacetime curved and flat indices are related by the vielbeins E^M_A, while for those in the world-volume we use $e^a{}_\mu = \partial_\mu X^M E^a_M$. Finally $\alpha, \beta, \ldots$ denote spinorial indices

[4] In certain cases, the p-forms constructed as bispinors contain extra matrices acting on the underlying pair of 10d supergravity spinors. For the sake of generality, we use the above generic form. In each particular case, the exact definition of the calibration form as a bispinor can be readily obtained from the supertranslation algebra.

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inequality becomes[5][6]

$$\mathcal{H} \geq \Phi|_\xi \,. \tag{2.5}$$

The condition that the object preserves supersymmetry in the background is that the above inequality is saturated.

Now note that for the case of a Dp-brane in a purely metric background, its energy comes just from the pure Nambu-Goto piece of its Dirac-Born-Infeld action, namely its energy density is basically

$$\mathcal{H} = \sqrt{-\det P[G]}\;\; \mathrm{vol}|_\xi \,, \tag{2.6}$$

where $P[G]$ represents the pullback of the ambient metric on the (spacelike) world-volume directions, and $vol|_\xi = d\sigma^1 \wedge \cdots d\sigma^p$ is the volume form in the world-volume of the p-extended object. Moreover, the pullback of the form Φ in (2.4) on the tangent plane to the brane can be shown to be

$$\begin{aligned} \Phi_{(p)}\big|_\xi &= \sqrt{-\det P[G]}\;\tilde{\Phi}_{(p)} \\ \text{with}\;\; (\tilde{\Phi})_{a_1\cdots a_p} &= \bar{\epsilon}\,\Gamma_{a_1\cdots a_p}\,\epsilon \end{aligned} \tag{2.7}$$

Then the above discussion directly leads to calibrations.[7] We have a closed form $\tilde{\Phi}_{(p)}$ that satisfies $\tilde{\Phi}_{(p)}|_\xi \leq vol_\xi$ and supersymmetry is preserved when the inequality is saturated. Thus supersymmetric branes wrap p-dimensional submanifolds calibrated with respect to $\tilde{\Phi}_{(p)}$.

Note that a calibrated submanifold is volume minimizing within its homology class. Hence it implies the familiar statement that a brane carrying no homology charge cannot have non-zero volume and preserve supersymmetry. We will see in latter sections that this property does not hold in more general backgrounds, with supersymmetric branes described by generalized calibrations.

2.2 Generalized calibrations

In the previous section we have seen that constructing Φ in (2.4) with a Killing spinor ϵ such that $\nabla_\nu \epsilon = 0$ guarantees that the form is a calibration and, in particular, that it is closed. However, in more general supersymmetric supergravity solutions, not only the metric but other backgrounds, like the dilaton and NSNS and RR field strength fluxes, are turned on. In such backgrounds, the covariant derivative of the Killing spinor is not zero, but is related to the additional fields. For a general background, the Killing spinor equation is obtained by imposing that the supersymmetric variations of the fermions vanish. In type

[5]More precisely, $\mathcal{H} \geq |\Phi_\xi|$, in order to account for extended objects of opposite charges, namely p- and anti-p extended objects. For concreteness in our examples we provide Φ_ξ without the absolute value, and implicitly assume a particular choice of orientation.

[6]Note that this is a local relation, involving the energy and charge densities. Once integrated over the world-volume, it relates the total energy and charge.

[7]An alternative viewpoint on calibrations vs. supersymmetry, which we will not exploit, is based on kappa symmetry; see, e.g. [29].

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IIB theory,[8] following the notation of [30, 31], they are given by

$$\delta_\epsilon \psi_M = \nabla_M \epsilon - \frac{1}{8} H_{MNP} \Gamma^{NP} \sigma^3 \epsilon + \frac{1}{16} e^\phi \sum_{n=1}^{5} \frac{1}{(2n-1)!} G^{(2n-1)}_{N_1 \cdots N_{2n-1}} \Gamma^{N_1 \cdots N_{2n-1}} \Gamma_M \lambda_n \epsilon \quad (2.8)$$

with ϕ the dilaton and $\lambda_n = \sigma^1$ $(i\sigma^2)$ for n even (odd). From the above equation one can obtain the expression of $\nabla_M \epsilon$ in terms of the background fields.

It is clear that, using the above Killing spinor to construct $\Phi_{M_1 \cdots M_p} = \bar{\epsilon} \Gamma_{M_1 \cdots M_p} \epsilon$, as in previous section, the resulting form is not closed. On the other hand, its exterior derivative will be related to the background fields. This points towards a generalization of the concept of calibration, valid for general supersymmetric backgrounds. Namely, a *generalized calibration* is a form containing a non-closed piece associated to the volume form of tangent planes, but involving additional pieces related to the additional backgrounds, and completing it to a closed form.

Such a generalized calibration will no longer calibrate minimal volume submanifolds, but minimal action world-volumes. A brane wrapping a generalized calibrated submanifold minimizes its action (which is no longer just the Nambu-Goto action but contains couplings to the additional background fields), and preserves some of the supersymmetry of the background. Some partial results on these generalized calibrations have been obtained in e.g. [18, 19]. Our aim is to derive and describe generalized calibrations more systematically, and including more general backgrounds and examples.

A possible strategy to construct generalized calibrations has been proposed in [32] in certain very supersymmetric backgrounds, based on ideas in [35]. The point is to consider the non-closed form $\bar{\epsilon} \Gamma_{M_1 \cdots M_p} \epsilon$, and to compute its exterior derivative in terms of the additional backgrounds, by using the Killing spinor equations. The result then *suggests* how to complete the initial non-closed form to a closed generalized calibration.

The strategy we follow is different, and far more systematic. As in previous section, we work at the level of the superalgebra, which for a general type IIB supergravity background can be written

$$\begin{aligned} \{Q_\alpha, Q_\beta\} = {} & (\Gamma^M)_{\alpha\beta} P_M + \sum_{n=1}^{5} (\Gamma_{M_1 \cdots M_{2n-1}})_{\alpha\beta} Z_{RR}^{M_1 \cdots M_{2n-1}} + (\Gamma_M)_{\alpha\beta} Y_{NS}^M + \\ & + (\Gamma_{M_1 \cdots M_9})_{\alpha\beta} Y_{NS}^{M_1 \cdots M_9} . \end{aligned} \quad (2.9)$$

This has the familiar structure, with one term involving the generator of translations, and central charges (to be made explicit below) related to possible extended objects in the background.

Once contracting with a Killing spinor ϵ, the right hand side becomes a sum of forms of different degrees. Again these forms, denoted Θ_{RR}, and Π_{NS}, characterize the different supersymmetric objects in the background. In the rest frame, we obtain the BPS inequality

$$\mathcal{H} \geq \sum_{n=1}^{5} \Theta^{RR}_{(2n-1)} \Big|_\xi + \Pi^{NS}_{(1)} \Big|_\xi + \Pi^{NS}_{(9)} \Big|_\xi \quad (2.10)$$

[8] In this section, for concreteness we center on type IIB theory, but the conclusions easily generalize to type IIA theory.

(where it is understood that in the right hand side only the contributions associated to the relevant object are present). The explicit expression of the calibrating forms Θ and Π in terms of the background fields is given by[9] [30, 31]

$$\begin{aligned}
\Theta^{RR}_{(2n-1)}\Big|_\xi &= i_K C_{(2n)} - e^{-\phi}\,\Phi_{(2n-1)}\big|_\xi - C_{(2n-2)}\wedge K|_\xi \\
\Pi^{NS}_{(1)}\Big|_\xi &= i_K B^{NS}_{(2)} + K|_\xi \\
\Pi^{NS}_{(9)}\Big|_\xi &= i_K B^{NS}_{(10)} + e^{-2\varphi}\,\Phi_{(9)}\big|_\xi
\end{aligned} \tag{2.11}$$

where the 1-form K was defined in (2.3), and $i_K C_{p+1}$ is a p-form defined by $(i_K C_{p+1})_{a_1\cdots a_p} = K^\rho (C_{p+1})_{\rho a_1\cdots a_p}$. The forms $\Phi_{(p)}$ are defined as in (2.4),[10] and $\Phi|_\xi$ denotes its pullback to tangent space indices.

An important remark about the above expression is that it is an equality between forms on the brane world-volume. Therefore, we will be careful to translate every piece to world-volume flat indices.

Similarly to the purely metric case, expression (2.10) allows us to interpret the different central extensions of the superalgebra as calibrating forms of the world-volume of extended objects. The saturation of the BPS inequality is the condition for supersymmetry, and implies that the energy minimizing and supersymmetric extended objects are those that wrap submanifolds calibrated by one (or a linear combination) of the forms $\Theta^{RR}_{(p)}$, $\Pi^{NS}_{(p)}$.

In order to understand the generalized calibration structure of these forms, let us center on the concrete case of a Dp-brane, for which the above central extension form reads

$$\Theta^{RR}_{(p)} = i_K C_{(p+1)} - e^{-\phi}\sqrt{-\det P[G]}\;\tilde{\Phi} - C_{(p-1)}\wedge K\,, \tag{2.12}$$

where $(\tilde{\Phi})_{a_1\cdots a_p} = \bar{\epsilon}\,\mathcal{P}_n\,\Gamma_{a_1\cdots a_p}\epsilon$ is the standard calibrating form, associated to the volume form. The factor of $\sqrt{-\det P[G]}$ is required for the translation of world-volume curved to flat indices (see expression (2.7)).

As pointed out at the beginning of the section, the closed calibration forms that enter in the superalgebra are not just a bilinear of spinors with a product of gamma matrices. Rather, as is explicit in the above expressions, there are extra terms that make the form closed. Equivalently, the closed calibrating form is not related to just the volume of submanifolds, but rather to the action of D-branes wrapped on them. The branes are supersymmetric when they saturate the BPS inequality, namely when their world-volume action is minimized and is given by the integral of the closed calibrating form. Therefore, each of the terms appearing in (2.11) should correspond to a world-volume coupling of the associated extended object. Let us see that this is indeed the case for the forms $\Theta^{RR}_{(p)}$ that calibrate Dp-branes.

[9] The reader may note that in our conventions (see the appendix) there is an extra minus sign in the second term compared with the corresponding expression in the references. Note that in those expressions there is a hidden minus sign in the contraction $K^\rho C_{\mu_1\ldots\mu_{2n-1}\rho} = -K^\rho C_{\rho\mu_1\ldots\mu_{2n-1}}$ that is absent in ours, so that the relative signs are the same.

[10] In this particular case, the IIB supertranslation algebra dictates that $\Phi_{2n-1} = \bar{\epsilon}\lambda_n\Gamma_{\mu_1\cdots\mu_{2n-1}}\epsilon$ with λ_n defined after (2.8), see footnote 4.

The first term in expression (2.11) is the ordinary Wess-Zumino term $\int_{Dp} C_{p+1}$, adapted to the fact that we are just considering the spatial directions of the brane. The second term realizes the ordinary volume term appearing in the DBI part of the action. Note that interestingly the dilaton prefactor in front of the volume term is also obtained.

Concerning the last term in $\Theta_{(p)}^{RR}$ its interpretation as a world-volume action term seems to be more involved. In the presence of a $C_{(p-1)}$ background, its field-strength acts as a source for the gauge field on a Dp-brane world-volume, $\int_{W_{p+1}} F_p \wedge A_1$, leading to a world-volume tadpole for the latter. Cancellation of the tadpole requires the presence of fundamental strings ending on the Dp-brane, leading to an additional contribution to the tension of the system. This is described precisely by the additional contribution to the central charge. This issue arises e.g. for D0-brane probes in the presence of the background created by D8-brane (or viceversa), and has been discussed in detail in [33] (see [34] for similar discussions in dual systems). It will not play any role in our examples, so we ignore it in further discussions.

We would like to finish this section with a remark. As we have just argued, in a general supergravity background the closed form that calibrates supersymmetric brane world-volumes is not only sensitive to the volume of the submanifold, but to a number of other terms (corresponding to additional couplings of the brane to the background). Hence, contrary to the intuition with standard calibrations, branes carrying no topological charge (e.g. wrapping homologically trivial cycles) may be supersymmetric even if they have non-zero volume. This beautiful feature of having stable and supersymmetric Dp-branes carrying no charges[11] leads to novel phenomena and allows for new model-building possibilities. We will explore some of them in the following sections.

2.3 Branes within branes

There are brane systems with more than one contribution to central charges. These include simple superpositions of different branes, preserving some common supersymmetry, but also bound states of branes. A familiar class of examples of the latter are obtained for D-branes with topologically non-trivial world-volume gauge bundles, or in the presence of NSNS B-fields. As follows from the above general discussion, the generalized calibration characterizing the supersymmetry properties of these bound states is obtained as a superposition of the corresponding central charges.

However, it will be useful for later sections to describe a more compact expression for these superpositions of central charges, for systems of D-branes with lower-dimensional induced charges. We consider the generalized calibration associated to a Dp-brane with a world-volume gauge field strength[12] F, and in the presence of a NSNS 2-form field B. Following the general discussion above, the generalized calibration should contain a piece describing the tension of the bound state, and a piece describing the coupling $i_K C_{(n)}$ for each of the charges present.

[11]Such branes have appeared in different contexts, for instance [36, 37, 38, 39].

[12]Calibrations for bound states of branes in the particular context of M-theory (and fivebranes in IIA) can be found in e.g. last reference of [20].

These two pieces can be immediately read from the structure of the general D-brane action with the above described backgrounds. Hence, the generalized calibration encoding the information of such bound states is given by

$$\Theta^{RR}_{(p)}\Big|_{\xi} = i_K \left(\sum_n C_{(n+1)} \wedge e^{(B+F)} \right)\Bigg|_{\xi} - e^{-\phi} \sqrt{-\det\left(P[g+B]+F\right)}\tilde{\Phi}_{(p)}\Big|_{\xi} \,, \qquad (2.13)$$

where $\tilde{\Phi}$ is defined after (2.12).

Clearly the first term describes a formal sum of the central charges associated to the coupling of lower-dimensional branes to the RR gauge potentials, and the second describes the tension of the bound state. This expression was already implicit in the description of the supersymmetry properties of branes in [40] from the κ-symmetry viewpoint.

The usefulness of the above expression will become more manifest in sections 4 and 5, where we consider D-branes in NSNS (and RR) flux backgrounds. However, it is important to notice a general drawback of the above expression: since it exploits induced charges and tensions on the D-brane volume, it over-emphasizes the role of the D-brane, whereas a true calibration is expected to be a form defined over spacetimes, which is ultimately pullbacked onto D-brane volumes. The subtlety is not relevant for situations with constant B-fields, but some effects may arise if non-trivial NSNS 3-form flux. Indeed, in some instances (see section 5.4) the above expression rather provides the actual action of a D-brane configuration, which only when minimized provides the value of the generalized calibration over the D-brane. For configurations attaining this minimum, the D-branes are generalized calibrated.

3. Some examples of generalized calibrated branes

3.1 Supergravity backgrounds

In this section we apply the above general discussion to some examples. A good testing ground for these ideas is provided by the supergravity backgrounds created by stacks of branes. For instance, consider the supergravity solution for a stack of N Dp-branes[13] (similar ideas may be discussed for other NS- or M-branes, or other supersymmetric backgrounds), given by [41]

$$\begin{aligned} ds^2 &= H(x)^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + H(x)^{1/2}dx^m dx^m \\ e^{2\phi} &= H(x)^{(3-p)/2} \\ C_{p+1} &= (\,H(x)^{-1} - 1\,)\, dx^0 \ldots dx^p\,. \end{aligned} \qquad (3.1)$$

Here $H(x)$ is a harmonic function in the transverse space. For a point-like source $H = 1+g_s N(\alpha')^{(7-p)/2}/r^{7-p}$, with $r^2 = \sum_m (x^m)^2$, but more generally some of such backgrounds may correspond/be mimicked by other kinds of sources, like fluxes, or distributions of branes and fluxes, by simply considering more general harmonic functions $H(x^m)$.

[13] As usual, this expression is valid for $p \leq 6$. For $p > 6$ the expression of the harmonic form changes, and the solution provides a local description of the spacetime, which is no longer asymptotically flat. However, our discussion can be applied to the latter cases as well.

We are interested in considering brane probes in these backgrounds. We may center on Dp'-brane probes for concreteness (with analogous results for NS-branes). The generalized calibration form associated to a probe Dp'-brane is given by (2.12)

$$\Theta^{RR}_{(p')} = i_K C_{(p'+1)} - e^{-\phi}\sqrt{-\det P[G]}\;\tilde{\Phi}_{(p')}\,, \tag{3.2}$$

where we have dropped the last term which, as anticipated, plays no rôle in the situations we are to consider. Recall that $P[G]$ represents the pullback of the ambient metric on the p' *spacelike* components of the world-volume. Also K, $\tilde{\Phi}_{(p')}$ are the 1- and p'-form defined as spinor bilinears with the Killing spinor of the supergravity background ϵ.

The Killing spinor preserved by a Dp'-brane background is of the form [31]

$$\epsilon = H^{-1/8}(r)\epsilon_0 \tag{3.3}$$

with ϵ_0 a constant spinor normalized to one, $\bar{\epsilon}_0\Gamma^0\epsilon_0{=}\epsilon_0^\dagger\epsilon_0 = 1$, and satisfying $\Gamma_{01\cdots p'}\epsilon_0{=}\epsilon_0$.

With this expression for the spinor, the second term of the calibration becomes

$$e^{-\phi}\sqrt{-\det P[G]}\,H^{-1/4}\;\tilde{\Phi}_0 \qquad \text{with} \qquad \left(\tilde{\Phi}_0\right)_{a_1\cdots a_{p'}} = \bar{\epsilon}_0\Gamma_{a_1\cdots a_{p'}}\epsilon_0\,. \tag{3.4}$$

Notice that the prefactor is responsible for this volume-related piece not being closed.

From expression (3.1), we have $H^{-1/4} = \sqrt{-g_{00}}$. Hence it can be absorbed in the square root of the determinant, to give the determinant of the induced metric along *all* $p'+1$ world-volume dimensions.

Concerning the first term, if we recast

$$K^M \;=\; \bar{\epsilon}\,\Gamma^M\,\epsilon \;=\; H^{-1/4}(r)\,e^M{}_a\,\bar{\epsilon}_0\,\Gamma^a\,\epsilon_0 \tag{3.5}$$

then the non-constant piece is factored out. The contraction $i_K C_{(p'+1)}$ then reads

$$\left(i_K C_{(p'+1)}\right)_{a_1\cdots a_{p'}} \;=\; H^{-1/4}(r)\,e^0{}_a\,\bar{\epsilon}_0\,\Gamma^a\,\epsilon_0\,C_{0a_1\ldots a_{p'}} \;=\; \bar{\epsilon}_0\,\Gamma^0\,\epsilon_0\,C_{0a_1\ldots a_{p'}}\,. \tag{3.6}$$

In total, we end up with the following general calibrating form for the Dp'-brane

$$\Theta^{RR}_{(p')} \;=\; C_{(p'+1)}\bar{\epsilon}_0\,\Gamma^0\,\epsilon_0 \;-\; e^{-\phi}\sqrt{-\det P[G]}\;\tilde{\Phi}_{(0)}\,, \tag{3.7}$$

where now $P[G]$ represents the pullback of the ambient metric on the complete $(p'+1)$-dimensional world-volume, and all Γ's have tangent space indices (and are thus constant). Note that the bispinorial factor in the first term is just the normalization condition for the spinor ϵ_0, that we have set to 1.

We would like to point out that the manipulations carried out to reach the final form of the generalized calibration can be similarly repeated for other brane-like supergravity solutions. Hence, examples like those is next section can be similarly constructed for generalized calibrations in supergravity backgrounds created by NS5-branes, M-branes, etc.

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3.2 Examples

We may consider several examples of calibrated and generalized calibrated submanifolds in these backgrounds, corresponding to supersymmetric branes.

3.2.1 An example with non-trivial metric and flux

Let us consider a simple example, which is a version of the generalized calibrations considered in [18], where both the volume of the cycle as well as a RR potential are involved in the calibration. Consider the above background for $p = 3$

$$\begin{aligned} ds^2 &= H(x)^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + H(x)^{1/2}dx^m dx^m \\ C_4 &= (H(x)^{-1} - 1)\, dx^0 \wedge \cdots \wedge dx^4 \end{aligned} \tag{3.8}$$

with constant dilaton. The particular case of $H(r) = \alpha'^2 g_s N/r^4$ corresponds to the maximally supersymmetric background $AdS_5 \times S^5$.

Consider a D3-brane spanning the direction 0123, and sitting at a value of the coordinates x^m. This submanifold in general does not minimize the volume, obtained from the familiar volume form induced from the metric

$$\sqrt{-\det P[G]}\, dx^0\, dx^1\, dx^2\, dx^3 \;=\; H(r)^{-1}dx^0\, dx^1\, dx^2\, dx^3 \tag{3.9}$$

unless $H(x) = 1$, i.e. a flat background. However, the D3-brane *is* supersymmetric in this background, since it is generalized calibrated with respect to the generalized calibration (3.7), corresponding to the background with flux, which takes the form

$$\begin{aligned} \Theta_{D3} &= C_4 - \sqrt{-\det P[G]}\, dx^0\, dx^1\, dx^2\, dx^3 \\ &= C_4 \;-\; H(x)^{-1}\, dx^0\, dx^1\, dx^2\, dx^3 \;=\; -dx^0\, dx^1\, dx^2\, dx^3\,. \end{aligned} \tag{3.10}$$

Notice that the generalized calibration completes the expression (3.9) with additional pieces, to yield a closed form. The D3-brane is supersymmetric since its action is given not just by its volume, but rather includes an 'electrostatic' energy due to its coupling to the background RR 4-form.

3.2.2 An exotic standard calibration

Consider the same background (3.8), now probed by a D5-brane spanning the directions 012456. Centering e.g. in the $AdS_5 \times S^5$ situation, written as

$$\begin{aligned} ds^2 &= \left(\frac{r}{R}\right)^2 \eta_{\mu\nu}dx^\mu dx^\nu + R^2\,\frac{dr^2}{r^2} \;+\; R^2\, d\Omega_5^2 \\ &= \left(\frac{r}{R}\right)^2 \eta_{\mu\nu}dx^\mu dx^\nu + R^2\,\frac{dr^2}{r^2} \;+\; R^2\,(\cos^2\theta\, d\Omega_2^2 + \sin^2\theta d\Omega_2'^2) \end{aligned} \tag{3.11}$$

the D5-brane is spanning an $AdS_4 \times S^2$ subspace. Although the D5-brane is wrapped on a trivial cycle in the internal manifold, it is supersymmetric. Indeed, applying the formula (3.7) to this situation (and noticing the RR field does not give any contribution),

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we see that the brane is calibrated by the form

$$\begin{aligned}\Theta_{D5} &= -\sqrt{-\det P[G]}\, dx^0\, dx^1\, dx^2\, dr\, d\mathrm{vol}_{S^2} \\ &= -r^2\, dx^0\, dx^1\, dx^2\, dr\, d\mathrm{vol}_{S^2} = -dx^0\, dx^1\, dx^2\, dx^4\, dx^5\, dx^6\,.\end{aligned} \tag{3.12}$$

Notice that the form is closed, and indeed corresponds to the volume form in flat space. In this sense, the complete submanifold spanned by the D5-brane minimizes its volume. On the other hand, regarding the configuration as a compactification to 5d, the D5-brane may seem able to decrease its volume (and hence its energy) by simply slipping off the S^2 in the S^5. The paradox is solved as follows. As discussed in [37], the scalar parameterizing the slipping is tachyonic in the AdS_4 non-compact directions of the D5-brane, but not tachyonic enough to violate the Breitenlohner-Friedmann bound [42], and hence does not lead to an instability. Hence, there is a key role played by the non-compact directions in the stability of the system. In other words, the process of slipping off the $\mathbf{S^2}$ requires exciting the spacetime profile of the tachyonic field, in such a way that the combined motion increases the energy. In this sense, the above configuration is volume minimizing, in agreement with it being calibrated. The result of having a stable brane wrapped on a trivial cycle is nevertheless surprising and interesting. It will play an important role in later sections. Indeed the above calibrated submanifolds will be exploited in section 6

3.2.3 Generalized hermitean calibrations

The above two kinds of calibrations (pure metric, and metric plus flux calibrations) may be combined. Consider again the background (3.8) and consider D3-brane probes wrapped on 2-cycles in the directions 2345. The volume form for such 2-cycles is

$$H(x^m)^{-1}\, dx^0\, dx^1\, dx^2\, dx^3 + dx^0\, dx^1\, dx^4\, dx^5\,. \tag{3.13}$$

In the $AdS_5 \times S^5$ case, the brane wraps an AdS_3 slice of the whole AdS_5 and a circle in the internal space. The second piece in the volume form precisely corresponds to the part of the D3-brane spanning the $AdS_3 \times S^1$ along 0145.

The total form is not closed, due to the first piece. On the other hand, supersymmetric branes must have world-volumes calibrated not with respect to (3.13), but with respect to

$$\begin{aligned}\Theta_{D3} &= -H(x)^{-1}\, dx^0\, dx^1\, dx^2\, dx^3 - dx^0\, dx^1\, dx^4\, dx^5 + C_4 \\ &= -dx^0\, dx^1\,(\, dx^2\, dx^3 + dx^4\, dx^5\,) = 2i\, dx^0\, dx^1\,(\, dz_1\, d\overline{z}_1 + dz_2\, d\overline{z}_2\,)\,,\end{aligned} \tag{3.14}$$

where we have introduced complex coordinates $z_1 = x^2 + ix^3$, $z_2 = x^4 + ix^5$. The generalized calibration contains additional pieces, providing a closed form.

The last expression makes it manifest that any D3-brane spanning 01 and a holomorphic 2-cycle in z_1, z_2 is generalized calibrated. These hermitean (as generalization of holomorphic) generalized calibrated submanifolds have been studied in [18]. The extension to other generalized calibrations (generalizing calibrated cycles in flat space or other special holonomy manifolds) can be carried out analogously [18], so we skip their discussion.

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3.2.4 An example with non-trivial metric and dilaton

It is interesting to describe generalized calibrated submanifolds in supersymmetric backgrounds with varying dilaton. Let us consider one such example. Consider the background (3.1) for $p = 5$, and a D3-brane probe along 0126. This corresponds to a supersymmetric probe, since it is generalized calibrated with respect to the calibration

$$\begin{aligned}\Theta_{D3} &= -e^{-\phi}\sqrt{-\det P[G]}\, dx^0\, dx^1\, dx^2\, dx^6 \\ &= -H(x)^{1/2}H(x)^{-1/2}\, dx^0\, dx^1\, dx^2\, dx^6 \;=\; -dx^0\, dx^1\, dx^2\, dx^6\,. \end{aligned} \tag{3.15}$$

The dilaton radial factor is canceled against the metric piece, determined by the number of directions with $H(r)^{\pm 1/2}$ factors. Clearly, many other examples are possible. For instance, one can easily derive that D3-branes spanning 01 and a 2-cycle in 2367 is generalized calibrated if the 2-cycle is holomorphic in the complex coordinates $z_1 = x^2 + ix^3$, $z_2 = x^6 + ix^7$.

3.2.5 An example with non-trivial metric, flux, and dilaton

Finally, let us describe a simple example where metric, dilaton, and RR fields enter in the generalized calibration. Consider the background (3.1) for $p = 4$, and consider a D4-brane probe along 01234. The D4-brane is supersymmetric, since it spans a generalized calibrated submanifold with respect to the form

$$\Theta_{D4} = C_5\, - e^{-\phi}\sqrt{-\det P[G]}\, dx^0\, dx^1\, dx^2\, dx^3\, dx^4 \;=\; -dx^0\, dx^1\, dx^2\, dx^3\, dx^4\,. \tag{3.16}$$

A similar calibration will appear in section 4.2.

4. Generalized calibrations in flux compactifications: a warm-up example

In this section we provide a simple example that illustrates that supersymmetric branes in flux compactifications[14] and compactifications on non-Calabi-Yau spaces (but with SU(3) structure) are generalized calibrated. A more general description will be provided in next section, but it is useful to consider a warm-up example first.

4.1 The type IIB D3-brane in a flux background

We are interested in considering type IIB string theory compactifications on Calabi-Yau threefolds, with NSNS and RR 3-form field strength fluxes, $\mathcal{H}_3$ and F_3, see [4, 5] for the basic description of the setup. For simplicity we may take a non-compact version, with topology $\mathbb{R}^5 \times \mathbf{S^1}$, with $\mathbb{R}^5$ parametrized by coordinates x^α, and $\mathbf{S^1}$ parametrized by a periodic coordinate x (on which we will eventually apply T-duality). We denote these six coordinates by x^m. Some additional directions beyond x could be considered to be compact, by imposing periodic identifications on the corresponding x^α's, as we will consider in certain

[14] It is worth noting that we center on supersymmetric fluxes of the B(ecker)-type (e.g. in [43]). It would be interesting to extend this work on generalized calibrations to other classes of supersymmetric fluxes like those appearing in [44]

instances (notice that if we consider all directions to be compact, thus describing a T^6, we will implicitly assume the introduction of orientifold O3-planes required for RR tadpole cancellation). Alternatively, the configuration can be regarded as a local model of more general situations.

Let us introduce RR and NS-NS 3-form fluxes F_3, $\mathcal{H}_3$ such that $G_3 = F_3 - \tau\mathcal{H}_3$ (with τ the IIB complex coupling) is imaginary self-dual (ISD).[15] The fluxes backreact on the background by inducing a warp factor for the metric and a non-vanishing RR 5-form field strength F_5. The supergravity solution for the configuration is of the black 3-brane form, due to the imaginary self-duality of the flux [4, 5]. We consider a flux distribution independent of x, so that the harmonic function H controlling the solution depends only on the x^α. The type IIB background is

$$\begin{aligned} ds^2_{IIB} &= H^{-1/2}(x^\alpha)\, ds^4_{4d} + H^{1/2}(x^\alpha)\, dx^m dx^m \\ C_4 &= (H^{-1}(x^\alpha) - 1) dvol_{0123} \\ C_2 &= (C_2)_{\alpha\beta} dx^\alpha dx^\beta + (C_2)_{x\alpha} dx dx^\alpha \Longrightarrow F_3 = (F_3)_{\alpha\beta\gamma} dx^\alpha dx^\beta dx^\gamma + (F_3)_{x\alpha\beta} dx dx^\alpha dx^\beta \\ \mathcal{B}_2 &= (\mathcal{B}_2)_{\alpha\beta} dx^\alpha dx^\beta + (\mathcal{B}_2)_{x\alpha} dx dx^\alpha \Longrightarrow \mathcal{H}_3 = (\mathcal{H}_3)_{\alpha\beta\gamma} dx^\alpha dx^\beta dx^\gamma + (\mathcal{H}_3)_{x\alpha\beta} dx dx^\alpha dx^\beta , \end{aligned} \tag{4.1}$$

where $dvol_{0123} = dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3$, and a convenient gauge has been chosen for $\mathcal{B}_2$, C_2. In principle we leave the explicit form of $H(x^\alpha)$ arbitrary, so that the configuration describes a general class of situations (including distant sources, like additional O3-planes or D3-branes).

Under some additional conditions (namely, that the flux G_3 is (2,1) and primitive), the above background is supersymmetric. Now consider we place on it a probe D3-brane along 0123. This preserves the same supersymmetry as the background, and indeed it corresponds to a brane on a generalized calibrated submanifold of 10d spacetime. The discussion is similar to that of a D3-brane in the presence of a D3-brane supergravity background, discussed in section 3.2.1. The D3-brane is generalized calibrated, with respect to the form

$$\Theta_{D3} = C_4 \, - \, H^{-1}(x^\alpha)\, dvol_{0123} \, . \tag{4.2}$$

The generalized calibrated nature of the D3-brane simply reflects the supersymmetric cancellation between the gravitational and RR 4-form interactions of the D3-brane with the background created by the fluxes.

The fact that such D3-branes are generalized calibrated is perhaps not surprising, since the class of the point is a homologically non-trivial 0-cycle in the Calabi-Yau, and the generalized calibration seemingly describes a mildly deformed version of the corresponding standard calibration on the underlying Calabi-Yau.[16] However, the fact that the D3-brane is generalized calibrated is really crucial, since D3-brane charge in the presence of fluxes is a torsion class in K-theory, and hence such D3-branes do not carry $\mathbb{Z}$-valued charges (or

[15] Note that in examples with additional compact dimensions, the fluxes along compact 3-cycles must be properly quantized.

[16] In fact, neglecting the flux backreaction, which is mimicked by taking constant H, the generalized calibration becomes a standard calibration.

even no charge at all, so that they are topologically trivial). Thus the only way in which the latter systems can be supersymmetric is to correspond to *generalized* calibrations. We will return to this point in section 4.3.

4.2 Generalized calibrations in non-CY backgrounds from T-duality

T-duality acts in a non-trivial way on backgrounds with 3-form fluxes, by transforming certain components of the NSNS flux into curvature contributions in the T-dual background. Hence, we can exploit T-duality to explore supersymmetric brane wrappings in these (non-Calabi-Yau) geometries.

Let us T-dualize the above background together with the probe D3-brane along the isometric internal direction x. In the discussion below, the IIA and IIB RR potential and field strengths are distinguished by their degree, while for the NSNS ones, we denote the IIB quantities by $(\mathcal{B}_2, \mathcal{H}_3)$ and the IIA quantities by (B_2, H_3). We also introduce the definitions in (A.1).

Using standard T-duality formulae (see the appendix), the resulting T-dual IIA metric is

$$ds^2_{IIA} = H^{-1/2}\,\left(ds^2_{0123} + (dx + g_{(x)})^2\right) + H^{1/2}\,dx^\alpha dx^\alpha \tag{4.3}$$

with

$$g_{(x)} = \frac{g^{IIA}_{x\alpha}}{g^{IIA}_{xx}}\,dx^\alpha = -\mathcal{B}_{x\alpha}\,dx^\alpha\,; \qquad \omega = -dg_{(x)} = -(\mathcal{H}_3)_{(x)\alpha\beta}\,dx^\alpha dx^\beta\,. \tag{4.4}$$

The NSNS 2-form and dilaton are

$$\begin{aligned} B_2 &= (\mathcal{B}_2)_{\alpha\beta}\,dx^\alpha dx^\beta \\ e^{\phi^{IIA}} &= \frac{e^{\phi^{IIB}}}{\sqrt{g^{IIB}_{xx}}} = H^{-1/4}\,. \end{aligned} \tag{4.5}$$

Concerning the RR potentials, we do not need to compute all of them in the dual side. As will become clear in what follows, the dual object to our original D3 is only sensible to the RR 5-form potential with components $(C_5)_{0123x}$, which turns out to be

$$(C_5)_{0123x} = [(C_4)_{0123}]_x = (H^{-1} - 1)\,dvol_{0123x}\,. \tag{4.6}$$

The resulting metric (4.3) has the same structure as that of a black 4-brane, but with a non-trivial twisting given by $g_{(x)}$. In the compact case, the compact manifold is no longer a T^6 but a twisted torus. In particular, the x-coordinate is non-trivially fibered over all those two tori parametrized by (x^α, x^β) such that $(\mathcal{H}_3)_{\alpha\beta x} \neq 0$. For each pair (x^α, x^β) the first Chern class of the non-trivial fibration is precisely given by the (quantized) integral of $\mathcal{H}_3$ in the T^3 spanned by x, x^α, x^β. This non-trivial fibration makes the topology of the resulting space quite peculiar, see, e.g. [7, 8]. For our purposes it is enough to mention that the dual manifold is neither Kähler nor complex (hence non Calabi-Yau) but nevertheless leads to supersymmetric flat Minkowski spacetime, as expected from the dual side.

Concerning the probe D3, its T-dual corresponds to a D4-brane spanning the 4d Minkowski directions, and wrapped on the circle fiber parametrized by x. The twisting on the metric implies that, in situations with compact dimensions, this 1-cycle corresponds to a torsion class. Consider a 3-cycle given by the circle fibration over a 2d space Σ on the base, such that $\int_\Sigma dg_{(x)} = K \in \mathbb{Z}$. Then the twisting implies that K times the class of the circle 1-cycle is trivial in homology, and hence K wrapped D4-branes can unwind and disappear.[17] We will come back to the unwinding process in section 4.3.

The fact that the 1-cycle parametrized by x is a torsion class in homology implies that D4-branes wrapped on them do not carry any $\mathbb{Z}$-valued charge. On these grounds, one would be tempted to propose that they are not BPS, and that K of these D4-branes dynamically decay to the vacuum. On the other hand, the T-duality with D3-branes suggests the D4-branes should be supersymmetric, and hence BPS and stable. The resolution of this seeming paradox lies in generalized calibrations. Indeed, the D4-branes wrap a generalized calibrated submanifold, and hence are BPS and stable, even if they do not carry any charge.

Following our general discussion, the generalized calibration for these D4-branes is

$$\Theta_{D4} = C_5 \; - \; e^{-\phi} \sqrt{-\det P[G]} \, d\,vol_{0123} \wedge dx \tag{4.7}$$

which upon straightforward computation reads

$$\Theta^{RR}_{D4}\big|_{D4} \; = \; -dx^0 \, dx^1 \, dx^2 \, dx^3 \, dx \tag{4.8}$$

showing that the D4-branes are indeed generalized calibrated.

4.3 Decay of D4-branes in trivial cycles, and T-dual of K-theory

We cannot refrain from making a small detour and discussing the process of unwinding of objects wrapped on the $\mathbf{S^1}$ fibers, like the above D4-branes. As discussed in footnote 17, a D4-brane wrapped over the $\mathbf{S^1}$, and sitting at a point on the base 2-cycle Σ, can be continuously deformed on the base until is has swept out the whole of the two-dimensional submanifold Σ, so that the winding is finally undone by the twisting of the fibration of $\mathbf{S^1}$ over Σ. This process violates D4-brane number in K units, where K is the first Chern class of the $\mathbf{S^1}$ fibration over Σ.

This process can also be regarded as mediated by a brane instanton (equivalently, there exist domain walls separating the two configurations, with and without D4-branes). The branes mediating the process are in fact D4-branes spanning the two-dimensional base Σ, times a codimension one 3-plane in 4d Minkowski space (so that they are to be interpreted as instantons or domain walls, depending on whether they are transverse to the time or space coordinates).

This description raises a number of interesting points. First, because of the twisting of the $\mathbf{S^1}$ fiber transverse to the instanton D4-brane, there is a world-volume topological charge of K units. The world-volume tadpole is canceled if there are exactly K D4-

[17] The argument for this is very similar to the unwinding of the $\mathbf{S^1}$ fiber in the Hopf fibration of $\mathbf{S^3}$ over $\mathbf{S^2}$ (this is similar to the case $K = 1$, for other values of K, one may just take the Lens space $\mathbf{S^3}/\mathbb{Z}_\mathbf{K}$). Namely, one can start with a D4-brane wrapped K-times over the $\mathbf{S^1}$, and sitting at a point on the base 2-cycle Σ. Next, deform it so that its projection on the base is a small circle, and then grow the latter until is has swept out the whole of Σ. By then the projection is a small circle 'on the opposite side' of Σ, and the twisting has managed to unwind the cycle in the x direction, so that it can safely shrink to zero size and disappear.

brane wrapped on the $\mathbf{S^1}$ fiber, and 'ending' on the instanton D4-brane[18] on Σ. This is exactly the feature required for the instanton to violate D4-brane winding number in K units.

A second interesting point is the following. In compactifications with non-CY geometries of the above kind, there is a RR tadpole contribution from the flux (T-dual of the familiar one of type IIB 3-form fluxes). Namely, due to the twisting of the geometry, there is a 10d coupling (see [11, section 7.1])

$$\int_{10d} dg_{(x)} \wedge F_{4(x)} \wedge C_5 \tag{4.9}$$

which leads to a tadpole for C_5, canceled against local sources. The instanton D4-brane we are studying is a source of RR 5-form C_5, namely the flux of F_4 through the 4-cycle dual to Σ jumps by one unit. This implies that in crossing the instanton, there is an increase in K units to the C_5-tadpole, arising from the interplay of the additional unit of F_4 with the K units of $dg_{(x)}$. This implies that the disappearance of the K D4-branes is accompanied by an increase of the flux contribution to the tadpole, so that tadpole cancellation is maintained.

It is interesting to relate the above process to the T-dual type IIB picture. Mapping objects and couplings in a simple way, the type IIB version of the process corresponds to the disappearance of K D3-branes via a brane instanton corresponding to a D5-brane wrapped on a 3-cycle with K units of $\mathcal{H}_3$ flux. This has been described in [45], and in [46] in the domain wall picture. The process also involves an increase of F_3-flux through the dual 3-cycle, so that the C_4-tadpole increase by K units, compensating the disappearance of D3-branes, and ensuring tadpole cancellation after the decay. In fact this process is the brane instanton process introduced in [14] to explain the decay of branes whose charge is non-trivial in homology, but trivial in K-theory. It is extremely amusing to see that in the type IIA language the whole discussion can be carried out purely in terms of homology, thus making it more geometrical and intuitive.

4.4 Generalized calibrated branes and mirror symmetry

In this section we intend to generalize the arguments in the previous sections to the case of 3 T-dualities, thus mirror symmetry [47], for a particular class of backgrounds, studied in [8]. We consider certain type IIB theory in flux backgrounds and carry out three T-dualities, such that each $\mathcal{H}_3$ component has at most one leg along the T-duality directions. The resulting geometries are half flat manifolds [8], i.e. a particular relatively simple class of manifolds with SU(3)-structure. Hence, they serve as a good testing ground to explore the possibilities of brane wrapping, an issue in which we show generalized calibrated subspaces are crucial.

Our aim is to follow the same strategy as above to find an example of those generalized calibrated branes. Namely, T-dualize a D3-brane in a supersymmetric flux background,

[18] More precisely, the wrapped and the instanton D4-branes join smoothly, in a geometry locally reproducing the smooth intersection $xy = \epsilon$.

along three isometric directions. The mirror picture is a D6-brane wrapping a generalized calibrated submanifold of a half-flat geometry.[19]

For this case, we slightly change the notation with respect to previous sections. We consider at least three compact dimensions, denoted x^α, along which we will T-dualize the configuration, and three additional ones, which may be non-compact, denoted y^α.

For concreteness, let us consider a type IIB background with ISD supersymmetric 3-form fluxes of the form

$$\begin{aligned} \mathcal{H}_3 &= (\mathcal{H}_3)_{y^\alpha y^\beta x^\gamma}\, dy^\alpha dy^\beta dx^\gamma \\ F_3 &= (F_3)_{x^\alpha x^\beta y^\gamma}\, dx^\alpha dx^\beta dy^\gamma\,. \end{aligned} \tag{4.10}$$

This is a restricted class of fluxes, which is ISD for $\tau = i$. Moreover, there are supersymmetric examples within it, e.g.

$$\begin{aligned} \mathcal{H}_3 &= -dx^1 dy^1 dy^3 \,+\, dx^2 dy^2 dy^3 \\ F_3 &= dx^1 dy^1 dx^3 - dx^2 dy^2 dx^3 \end{aligned} \tag{4.11}$$

which in fact preserves $N = 2$ supersymmetry.

Since we take the coordinates $\vec{x}$ to be isometric, the most appropriate gauge to chose is:

$$\begin{aligned} \mathcal{B}_2 &= (\mathcal{H}_3)_{y^\alpha y^\beta x^\gamma}\, y^\alpha\, dy^\beta dx^\gamma \\ C_2 &= (F_3)_{x^\alpha x^\beta y^\gamma}\, y^\gamma\, dx^\alpha dx^\beta\,. \end{aligned} \tag{4.12}$$

Again the presence of the fluxes generates a black 3-brane background of the form

$$\begin{aligned} ds^2 &= H^{-\frac{1}{2}}\, ds^2_{4d} \,+\, H^{\frac{1}{2}}\, \left(d\vec{x}^2 \,+\, d\vec{y}^2 \right) \\ C_4 &= (H^{-1} - 1)\, dvol_{0123} \end{aligned} \tag{4.13}$$

where we take the harmonic function $H = H(\vec{y})$ to be independent of the x^α. By T-dualizing along the three coordinates x^α as in the previous section, the resulting metric and dilaton in the IIA side read

$$\begin{aligned} ds^2 &= H^{-\frac{1}{2}} \left(ds^2_{4d} \,+\, \sum_{\alpha=1}^{3} (\, dx^\alpha + g_{(x^\alpha)}\,)^2 \right) \,+\, H^{\frac{1}{2}}\, d\vec{y}^2 \\ e^\phi &= H^{-3/4}\,, \end{aligned} \tag{4.14}$$

where we defined $g_{(x^\alpha)}$ in terms of the original fields as in (4.4), namely $g_{(x^\alpha)} = -\mathcal{B}_{x^\alpha y^\beta} dy^\beta$. In the dual side, for the particular choice of $\mathcal{H}_3$ we have considered, no B-field component survives.

Following [8], this realization of mirror symmetry should yield a half-flat manifold in the T-dual. This can be verified by defining the tangent complex 1-forms

$$e^\alpha = H^{-1/4} \left(dx^\alpha + g_{(x^\alpha)} \right) + i H^{1/4}\, dy^\alpha \tag{4.15}$$

[19] An example of D6-brane wrapping a 3-cycle on a (non half-flat) SU(3)-structure manifold is discussed in [48]

and constructing the 2- and 3-forms

$$J = -\frac{i}{2}\left(e_1 \wedge \overline{e_1} + e_2 \wedge \overline{e_2} + e_3 \wedge \overline{e_3}\right) ; \qquad \Omega = e_1 \wedge e_2 \wedge e_3 . \tag{4.16}$$

These are globally defined but not closed. One easily checks that they satisfy the conditions

$$d(J \wedge J) = 0 ; \qquad d(\mathrm{Im}\Omega) = 0 \tag{4.17}$$

which are the conditions defining a half-flat manifold.

Concerning the dualization of the RR gauge potentials, we need not carry it out in full generality for our purpose. The only component relevant for our discussion below is

$$(C_7)_{0123x^1x^2x^3} = (H^{-1} - 1)\, dvol_{0123} \wedge dx^1 \wedge dx^2 \wedge dx^3 . \tag{4.18}$$

We are interested in the mirror of a IIB D3-brane, namely a D6 wrapping $0123x^1x^2x^3$. This shares many of the features of the D4-brane studied in previous sections. For instance, such D6-branes correspond to torsion classes in the 3-homology of the half-flat manifold, and hence carry no $\mathbb{Z}$-valued charges. They are nevertheless stable and supersymmetric, since they are generalized calibrated, with respect to the form

$$\Theta^{RR}_{(6)} = C_7 \,-\, e^{-\phi}\sqrt{-\det P[G]}\, dvol_{123} \wedge dx^1 \wedge dx^2 \wedge dx^3 \tag{4.19}$$

which more explicitly reads

$$\begin{aligned}\Theta^{RR}_{(6)} &= \left(-H^{3/4}\sqrt{(H^{-\frac{1}{2}})^7} + (H^{-1} - 1)\right) dvol_{123} \wedge dx^1 \wedge dx^2 \wedge dx^3 \\ &= -dvol_{123} \wedge dx^1 dx^2 dx^3 \end{aligned} \tag{4.20}$$

Given the nice geometric structure of half-flat manifolds as SU(3)-structure manifolds, it is natural to propose a characterization of generalized calibrated submanifolds in terms of the natural forms in the geometry, J and Ω. We indeed make such proposal, and provide evidence for it, in next section.

5. Generalized calibrations in compactifications with fluxes and on SU(3)-structure manifolds

5.1 The proposal

In this section we propose a characterization of generalized calibrated submanifolds in flux and non-CY compactifications, in analogy with standard calibrated submanifolds is standard Calabi-Yau compactifications.[20]

[20]For extensive work on the relation between generalized calibrations and G-structures, mainly in the context of M-theory and fivebranes see [20, 21]

Supersymmetric cycles in Calabi-Yau compactifications are nicely characterized in terms of the Kahler 2-form J and the holomorphic 3-form Ω of the geometry. They fall in two classes, holomorphic cycles and special lagrangian 3-cycles, defined by familiar conditions, see below.

Supersymmetric compactifications with 3-form fluxes, and on non-CY geometries, are generalizations of CY compactifications (see [8, 9] for a more extensive discussion). These backgrounds have a Killing spinor ϵ, namely a globally defined spinor which is not covariantly constant with respect to the spin connection related to the metric, but is covariantly constant with respect to a connection with torsion. This implies that the structure group of the tangent bundle is SU(3), hence the name SU(3)-structure compactifications. The invariant spinor may be used to construct a globally defined 2-form J and 3-form Ω, which in contrast with the CY case, are not closed. Their exterior derivatives are classified by the torsion classes of the connection, and are related to the non-metric part of the background (see e.g. [8]).

The last statements are very suggestive of a relation with generalized calibrations. Indeed we suggest that generalized calibrated submanifolds in SU(3)-structure manifolds are characterized, in analogy with the Calabi-Yau case, using the 3-form Ω and a complexified version of J, namely e^{J+iB}. The geometrical structure suggests the definitions of

i) (generalized) holomorphic cycles, characterized by the conditions

$$\Omega|_\xi = 0\,; \qquad e^{J+iB}|_\xi = vol_\xi \tag{5.1}$$

ii) (generalized) special lagrangian 3-cycles, characterized by

$$(J+iB)|_\xi = 0\,; \qquad \mathrm{Im}\Omega|_\xi = 0 \quad ; \quad \mathrm{Re}\Omega|_\xi = vol_\xi\,. \tag{5.2}$$

The generalized structure of these calibrated cycles lies not in the conditions, but in the property that the forms $J+iB$, Ω are not closed. In order to write the corresponding generalized calibrations, it is convenient to factorize the volume term into a piece from the non-compact spacetime and a piece from the internal submanifold volume, which can be recast using (5.1), (5.2). Denoting $\omega|_\xi = e^{J+iB}|_\xi$ for holomorphic and $\omega|_\xi = \mathrm{Re}\Omega|_\xi$ for special lagrangian submanifolds, the generalized calibration, for D-branes, may be written

$$\Theta_{(Dp)} = \Theta_{CS} - e^{-\phi}\sqrt{-\det P[G]}\,\tilde{\Phi}_0 \wedge \omega|_\xi\,, \tag{5.3}$$

where Θ_{CS} contains the terms associated to CS couplings, and $\tilde{\Phi}_0$ refers to the non-compact piece of the world-volume.

This proposal could presumably be verified by using the expression of the supersymmetry variations of general supergravity backgrounds in terms of Ω and e^{J+iB} [9]. Notice also that the exchange of both forms under mirror symmetry [9] suggest a mirror exchange of both kinds of branes, generalizing the familiar map for D-branes on Calabi-Yau spaces. We leave these very interesting exploration for future research. Here we limit ourselves to verifying the above proposal in a set of examples, obtained from (generalized holomorphic) D-branes in 3-form flux backgrounds, and (generalized special lagrangian) D6-branes in half-flat geometries (which for suitable NSNS fluxes are related by mirror symmetry [8]). Still this leads to interesting results, like the stabilization of D7-brane moduli by 3-form fluxes. It would be nice to demonstrate the above relation in more generality, and for other kinds of branes.

5.2 The D3-brane and its mirror, revisited

One first check of the proposal is to recover the results of section 4.4 from the above prescription. The case of D3-branes in a general supersymmetric 3-form flux background is straightforward. They correspond to generalized holomorphic calibrated submanifolds, because they satisfy $\Omega|_{\xi}=0$ since D3-branes span just a point in the internal space. They are supersymmetric, and the generalized calibration contains a volume term related to the restriction of $e^{J+i\mathcal{B}}$, in a trivial way.

A more interesting case is provided by the generalized calibration structure of its mirror D6-brane, in the half-flat background (4.14). It wraps a 3-submanifold of the internal space, which we claim is a generalized special lagrangian calibrated submanifold, according to our description above. Indeed, using the complex orthogonal frames (4.15), the 2- and 3-forms J and Ω (4.16) read

$$J=-\frac{i}{2}\left(e_1\wedge\overline{e_1}+e_2\wedge\overline{e_2}+e_3\wedge\overline{e_3}\right)=\sum_{i=1}^{3}dy^i\wedge\left(dx^i+g_{(x^i)}\right) \tag{5.4}$$

$$\begin{aligned}\Omega &= e_1\wedge e_2\wedge e_3\\ &=\Big\{H^{-3/4}(dx^1+g_{(x^1)})\wedge(dx^2+g_{(x^2)})\wedge(dx^3+g_{(x^3)})-H^{1/4}dy^1\wedge dy^2\wedge(dx^3+g_{(x^3)})-\\ &\quad -H^{1/4}dy^1\wedge(dx^2+g_{(x^2)})\wedge dy^3-H^{1/4}(dx^1+g_{(x^1)})\wedge dy^2\wedge dy^3\Big\}+\\ &\quad +i\Big\{H^{-1/4}(dx^1+g_{(x^1)})\wedge(dx^2+g_{(x^2)})\wedge dy^3+H^{-1/4}(dx^1+g_{(x^1)})\wedge dy^2\wedge(dx^3+g_{(x^3)})+\\ &\quad +H^{-1/4}dy^1\wedge(dx^2+g_{(x^2)})\wedge(dx^3+g_{(x^3)})-H^{3/4}dy^1\wedge dy^2\wedge dy^3\Big\}\,.\end{aligned} \tag{5.5}$$

We can readily check that the conditions (5.2) are automatically satisfied for the D6-brane along x^1, x^2, x^3, since all components of J and $\mathrm{Im}\Omega$ have at least one leg along dy^i.

Moreover, according to our proposal, the D6-brane should be calibrated with respect to a generalized calibration, with the volume piece related to $\mathrm{Re}\Omega$. Indeed it is straightforward to check that the generalized calibration (4.19) can be written

$$\Theta^{RR}_{(6)}=C_7-e^{-\phi}\sqrt{\det P[G]_{0123}}\,dvol_{123}\wedge\mathrm{Re}\Omega\,. \tag{5.6}$$

5.3 D7 branes in the presence of $\mathcal{N}=2$ G_3 fluxes.

Let us now turn to some new examples. Consider IIB theory on T^6 or a partially decompactified version thereof, in the background generated by the 3-form fluxes

$$\begin{aligned}\mathcal{H}_3 &= -dx^1dy^1dy^3+dx^2dy^2dy^3\\ F_3 &= dx^1dy^1dx^3-dx^2dy^2dx^3\end{aligned} \tag{5.7}$$

which are ISD and in fact $N = 2$ supersymmetric for $\tau = i$. Using the complex coordinates $z^j = x^j + iy^j$, we have

$$G_3 = \frac{i}{2}\,(\,dz_1\,d\bar{z}_1\,dz_3 \;-\; dz_2\,d\bar{z}_2\,dz_3\,)\,. \tag{5.8}$$

The associated potentials, in a suitable gauge, read

$$\begin{aligned} \mathcal{B}_2 &= -y^3\;dx^1dy^1 + y^3\;dx^2dy^2 \\ C_2 &= x^3\;dx^1dy^1 - x^3\;dx^2dy^2\,. \end{aligned} \tag{5.9}$$

They generate a black 3-brane background

$$\begin{aligned} ds^2 &= H^{-\frac{1}{2}}(ds^2_{4d}) + H^{\frac{1}{2}}\left((d\vec{x})^2 + (d\vec{y})^2\right) \\ C_4 &= (H^{-1}-1)dvol_{0123}\,. \end{aligned} \tag{5.10}$$

Concerning the dual RR potentials C_8, C_6, and straightforward computation shows that they vanish in our particular background (see the appendix for conventions on generalized field strengths, etc).

Let us consider a D7-brane spanning 0123 and the internal directions $z_1, \bar{z}_1, z_2, \bar{z}_2$, and located at a fixed coordinate z_3. Since this brane is wrapped on a holomorphic 4-cycle of the underlying Calabi-Yau, it is expected to be supersymmetric. Indeed, in the remainder of this section, we show that it is generalized calibrated, and that it can be regarded as a generalized holomorphic brane, in the description of section 5.1.

Since the background contains non-zero RR fluxes and NSNS $\mathcal{B}$-field, the appropriate generalized calibration for this case is the one in section 2.3. For our D7-brane the expression (2.13) reads

$$\Theta^{RR}_{(7)}\Big|_{\xi} = i_K\left(C_8 + C_6\wedge\mathcal{B}_2 + \frac{1}{2}C_4\wedge\mathcal{B}_2\wedge\mathcal{B}_2 + \cdots\right) + e^{-\phi}\sqrt{\det\left(P[g+\mathcal{B}]\right)}\,\tilde{\Phi}^{(7)}_{(0)}\Big|_{\xi} \tag{5.11}$$

Note that the relative sign of the second term has changed with respect to that in former sections. The reason is that we consider a D7 wrapping the holomorphic cycle with volume form $dz^1\wedge d\bar{z}^1\wedge dz^2\wedge d\bar{z}^2$. The volume form we are using in real coordinates $dx^1\wedge dx^2\wedge dy^1\wedge dy^2$, which has opposite orientation, hence yielding an extra sign.

The determinant of the pullback of $g+\mathcal{B}$, and the contribution from the Chern-Simons coupling, are easily computed to be

$$\begin{aligned} \sqrt{\det P[g+\mathcal{B}]} &= H^{-1}\,(\,H + (y^3)^2\,) \\ C_4\wedge\mathcal{B}_2\wedge\mathcal{B}_2 &= -2H^{-1}\;(y^3)^2 dvol_{0123}\wedge dx^1dy^1dx^2dy^2 \end{aligned} \tag{5.12}$$

We then have

$$\begin{aligned} \Theta^{RR}_{(7)}\Big|_{D7} &= \left(-H^{-1}(y^3)^2 + H^{-1}\left(H+(y^3)^2\right)\right) dvol_{0123}\wedge dx^1dy^1dx^2dy^2 \\ &= dvol_{0123}\wedge dx^1dy^1dx^2dy^2\,. \end{aligned} \tag{5.13}$$

Hence the expression is closed and corresponds to a generalized calibration, and the D7-brane is generalized calibrated. Notice that this is so for any value of the D7-brane transverse coordinate z_3. In section 5.4 we will describe an example where the D7-brane is generalized calibrated (i.e. supersymmetric) only at a fixed transverse position.

Let us now describe the above generalized calibrated submanifold in terms of the conditions in section 5.1. We introduce a set of complex vielbeins

$$e_i = H^{1/4}(dx^i + idy^i)\,. \tag{5.14}$$

The pullback of the 3-form $\Omega = e_1 \wedge e_2 \wedge e_3$ on the D7-brane is automatically zero. Hence, the D7-brane spans a generalized holomorphic calibrated 4-submanifold. Moreover, the generalized calibration should contain a volume piece associated to $e^{J+i\mathcal{B}}$, or rather its relevant piece for a 4-submanifold, $\frac{1}{2}(J+i\mathcal{B})^2$. Indeed, using (5.9) and (5.14), we have

$$\begin{aligned}
J+i\mathcal{B} &= -\frac{i}{2}\sum_i e_i \wedge \bar{e}_i + i\mathcal{B} \\
&= H^{1/2}\left(dx^1\wedge dy^1 + dx^2\wedge dy^2 + dx^3 \wedge dy^3\right) + \\
&\quad + i\left(-y^3 dx^1\wedge dy^1 + y^3 dx^2 \wedge dy^2\right) \\
(J+i\mathcal{B})\wedge(J+i\mathcal{B})|_{D7} &= 2\left(H - \mathcal{B}_{x_1y_1}\mathcal{B}_{x_2y_2}\right) dx^1\wedge dy^1 \wedge dx^2 \wedge dy^2 \\
&= 2\left(H + (y^3)^2\right) dx^1\wedge dy^1\wedge dx^2\wedge dy^2\,.
\end{aligned} \tag{5.15}$$

This indeed reproduces (once the factor from the non-compact directions is considered) the volume piece of the generalized calibration (5.12), which can thus be recast as

$$\Theta^{RR}_{(7)}\Big|_\xi = i_K\left(\frac{1}{2}C_4\wedge\mathcal{B}_2\wedge\mathcal{B}_2\right) + e^{-\phi}\sqrt{\det P[G]_{0123}}\,\frac{1}{2}(J+i\mathcal{B})\wedge(J+i\mathcal{B})\tilde{\Phi}_0\Big|_\xi\,. \tag{5.16}$$

Let us consider the mirror version of this system, by carrying out three T-dualities along the x^i coordinates. Using the results in section 4.4, the NSNS background in the IIA side is

$$\begin{aligned}
ds^2 &= H^{-1/2}\left(ds^2_{0123} + (dx^1 + g_{(x^1)})^2 + (dx^2 + g_{(x^2)})^2 + (dx^3)^2\right) + \\
&\quad + H^{1/2}\left((dy_1)^2 + (dy_2)^2 + (dy^3)^2\right) \\
e^{-\phi} &= H^{3/4}
\end{aligned} \tag{5.17}$$

with $B_2 = 0$ and $g_{(x^i)} \equiv -\mathcal{B}_{x^iy^\alpha}dy^\alpha$. In our case

$$\begin{aligned}
g_{(x^1)} &= y^3\ dy^1 \\
g_{(x^2)} &= -y^3\ dy^2 \\
g_{(x^3)} &= 0\,.
\end{aligned} \tag{5.18}$$

Concerning the RR potentials, the only component relevant for our purposes below is $(C_7)_{0123y^1y^2x^3}$ which, from our original configuration and using the T-duality formulas in the appendix, turns out to be:

$$(C_7)_{0123y^1y^2x^3} = -H^{-1}(y^3)^2 dvol_{0123}dy^1dy^2dx^3\,. \tag{5.19}$$

We are interested in describing the mirror of the original D7-brane, which is given by a D6-brane along the coordinates 0123 and y^1, y^2, x^3. Since there is no B-field present, the generalized calibration is provided by (2.12), namely

$$\Theta^{RR}_{(6)}\Big|_{D6} = i_K(C_7) + e^{-\phi}\sqrt{\det P[G]}dvol_{123} \wedge dy^1 \wedge dy^2 \wedge dx^3 \,, \tag{5.20}$$

where $P[G]$ is the pullback of the metric onto the full (spacetime) world-volume directions. Notice that this dual realization shows that expression (2.13) is the generalized calibration to be used in the original IIB case.

By considering the pullbacked metric

$$ds^2\big|_{D6} = H^{-1/2}\left(ds^2_{0123} + (y^3)^2\,(dy^1)^2 + (y^3)^2\,(dy^2)^2 + (dx^3)^2\right) + H^{\frac{1}{2}}\left((dy^1)^2 + (dy^2)^2\right) \tag{5.21}$$

whose determinant square root reads

$$\sqrt{-\det P[G]} = H^{-7/4}(H + (y^3)^2) \tag{5.22}$$

we get the result

$$\begin{aligned}\Theta^{RR}_{6}\big|_{D6} &= \left(-H^{-1}\,(y^3)^2 + H^{-1}\left(H + (y^3)^2\right)\right) dvol_{0123} \wedge dy^1 dy^2 dx^3 \\ &= dvol_{0123} \wedge dy^1 dy^2 dx^3 \,.\end{aligned} \tag{5.23}$$

Hence the D6-brane is calibrated in this half-flat manifold, as expected from the mirror image. Notice that the result obtained in IIA with the generalized calibration (2.12) reproduces the mirror result in IIB with the generalized calibration (2.13). This lends additional support for the proposal to include NSNS fields (and world-volume gauge couplings) in section 2.3.

Finally, we would like to describe this D6-brane as a generalized special lagrangian calibration. Introducing a complex orthonormal frame, and defining J and Ω as usual, we again find that the two conditions in (5.2) are automatically satisfied.

Hence the 3-cycle satisfies the generalized special lagrangian calibration conditions. Moreover, the pullback of $\mathrm{Re}\Omega$ should provide the volume piece in the generalized calibration. Indeed, we have

$$\mathrm{Re}\Omega|_\xi = -H^{-3/4}(H + (y^3)^2)\, dy^1 \wedge dy^2 \wedge dx^3 \tag{5.24}$$

so that (5.20) may be rewritten as

$$\Theta^{RR}_{(6)} = i_K(C_7) - e^{-\phi}\sqrt{\det P[G]_{0123}}\, dvol_{123} \wedge \mathrm{Re}(\Omega)\,. \tag{5.25}$$

We hope these examples suffice to illustrate how the proposal in section 5.1 can be used to characterize supersymmetric wrapped branes in flux backgrounds and half-flat manifold. We also expect it to have some more general validity in the complete class of SU(3)-structure manifolds. Unfortunately, lack of explicit examples makes it difficult to carry further tests of this stronger form of the proposal.

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5.4 Generalized calibrations and fixing D-brane moduli

In this section we describe an example of a brane which is generalized calibrated only at a particular position of its transverse space. In the type IIB picture of branes in flux backgrounds, this reproduces how fluxes fix moduli of D-branes wrapped on non-rigid cycles of the underlying Calabi-Yau.

Consider a type IIB $(2,1)$ and primitive G_3 flux background

$$G_3 = dz_1 dz_2 d\overline{z}_3 \,. \tag{5.26}$$

In real coordinates, for $\tau = i$, the corresponding RR and NSNS 3-form fluxes, and 2-form potentials, read

$$\begin{aligned} F_3 &= dx^1dx^2dx^3 \,+\, dx^1dy^2dy^3 \,-\, dy^1dy^2dx^3 \,+\, dy^1dx^2dy^3 \\ \mathcal{H}_3 &= dx^1dx^2dy^3 \,-\, dx^1dy^2dx^3 \,-\, dy^1dx^2dx^3 \,-\, dy^1dy^2dy^3 \\ C_2 &= x^3dx^1dx^2 \,+\, y^3dx^1dy^2 \,-\, x^3dy^1dy^2 \,+\, y^3dy^1dx^2 \\ \mathcal{B}_2 &= y^3dx^1dx^2 \,-\, x^3dx^1dy^2 \,-\, x^3dy^1dx^2 \,-\, y^3dy^1dy^2 \,. \end{aligned} \tag{5.27}$$

As usual, these generate a black 3-brane background of the form:

$$\begin{aligned} ds^2 &= H^{-\frac{1}{2}}(ds^2_{0123}) + H^{\frac{1}{2}}\left((d\vec{x})^2 + (d\vec{y})^2\right) \\ C_4 &= H^{-1} dvol_{0123} \end{aligned} \tag{5.28}$$

(where we dropped the constant term in C_4 for simplicity).

Consider a D7-brane spanning 0123 and z_1, $\overline{z}_1$, z_2, $\overline{z}_2$, and located at a point in the z_3 coordinate. Our purpose is to understand the generalized calibrated structure of such D7-branes.

In principle, one can work as in previous sections, with the expression (5.11) for the calibrating form for a D7-brane in a 3-form flux background. Instead, we follow the simpler approach introduced in previous section, of describing the generalized calibration conditions directly from J and Ω.[21] The D7-brane probe wraps a generalized holomorphic 4-cycle of the internal space, corresponding to the fact that it satisfies $\Omega|_\xi = 0$. Moreover, it must be generalized calibrated when it minimizes its action

$$\begin{aligned} \Theta^{RR}_{(7)} = {} & i_K\left(C_8 + C_6\wedge\mathcal{B}_2 + \frac{1}{2}C_4\wedge\mathcal{B}_2\wedge\mathcal{B}_2 + \cdots\right) + \\ & + e^{-\phi}\sqrt{\det P[G]_{123}}\,H^{-1/4} dvol_{123}\wedge\frac{1}{2}\,(J+i\mathcal{B})\wedge(J+i\mathcal{B})|_\xi \;, \end{aligned} \tag{5.29}$$

where again the sign of the second term has changed with respect to the original formulation due to orientation conventions.

We have

$$\begin{aligned} J + i\mathcal{B} = {} & -\frac{i}{2}\sum_i e_i\wedge\overline{e}_i + i\mathcal{B} \\ = {} & H^{1/2}\left(dx^1\wedge dy^1 + dx^2\wedge dy^2 + dx^3\wedge dy^3\right) + \\ & + i(y^3dx^1\wedge dx^2 \,-\, x^3dx^1\wedge dy^2 \,-\, x^3dy^1\wedge dx^2 \,-\, y^3dy^1\wedge dy^2) \end{aligned} \tag{5.30}$$

[21] We have indeed verified that the explicit construction of the calibrating form leads to equivalent results.

which leads to

$$(J + i\mathcal{B}) \wedge (J + i\mathcal{B})|_\xi = 2\left(H + |z_3|^2\right). \tag{5.31}$$

Concerning the Chern-Simons part of the calibration, it involves the C_6 and C_8 RR dual gauge potentials. However, one can easily compute that they again vanish in our case, so that the only relevant Chern-Simons coupling is:

$$C_4 \wedge \mathcal{B}_2 \wedge \mathcal{B}_2 = 2H^{-1}|z_3|^2 dvol_{0123} \wedge dx^1 \wedge dy^1 \wedge dx^2 \wedge dy^2 \tag{5.32}$$

In total, expression (5.29) reads

$$\begin{aligned} \Theta^{RR}_{(7)}\Big|_\xi &= \left(H^{-1}|z_3|^2 + H^{-1}(H + |z_3|^2)\right) dvol_{0123} \wedge dx^1 \wedge dy^1 \wedge dx^2 \wedge dy^2 \\ &= \left(1 + 2H^{-1}|z_3|^2\right) dvol_{0123} \wedge dx^1 \wedge dy^1 \wedge dx^2 \wedge dy^2 \end{aligned} \tag{5.33}$$

Notice that this expression is not closed, in agreement with our comments that expression (2.13) in general provides the action for a D-brane configuration rather than the generalized calibration. The action is minimized for $z_3 = 0$, where it takes the value

$$\Theta^{RR}_{(7)}\Big|_\xi = dvol_{0123} \wedge dx^1 \wedge dy^1 \wedge dx^2 \wedge dy^2 \tag{5.34}$$

This agrees with the restriction to the world-volume of a closed generalized calibrated form. Hence, interestingly the D7 is generalized calibrated only if it is located at $z_3 = 0$ in transverse space. This simply means that in the system at hand there is a non-trivial superpotential for the D7-brane geometric moduli φ_3 parameterizing its position in z_3, such that the only supersymmetric minimum is at $\varphi_3 = 0$.

This fits nicely with a similar result obtained from alternative approaches, where the effect of fluxes on D7-branes is computed directly. Indeed, a quadratic superpotential arises for D7-brane moduli in certain supersymmetric fluxes, as shown in [17] from the D7-brane world-volume perspective, and in [49] from the F-theory perspective. This result is important in that it modifies the infrared dynamics of D7-brane gauge theories in flux backgrounds. In particular, as already mentioned in [23], flux effects generate masses for D7-brane matter, even in supersymmetric situations, hence leading to non-perturbative superpotentials from gaugino condensation, which provide a source of stabilization for Kahler moduli. We hope that the techniques developed here (as well as in complementary approaches) help in rendering this effect tractable, so that it can be quantitatively included in further discussions of the construction of models with full moduli stabilization.

6. Model building with NS-branes

We would like to mention another possible application of branes wrapped on generalized calibrated submanifolds. In particular, one can take advantage of the fact that such configurations can be supersymmetric even if the branes carry no charges. This can be employed in model building since it allows to avoid the (sometimes very constraining) tadpole cancel-

	0	1	2	3	4	5	6	7	8	9
NS	–	–	–	x	–	–	–	x	x	x
NS'	–	–	–	x	–	x	x	x	–	–
D5	–	–	–	x	–	–	x	x	–	x

Table 1: Configuration of D- and NS-branes. A line denotes the brane spans the corresponding coordinate, while a cross indicates the brane is transverse to it.

lation conditions present in the familiar situations with charged branes. It may be argued that branes carrying no charge in homology are unable to lead to chiral fermions; we will see below how net chirality may be achieved even for branes in homologically trivial cycles.

In addition, it allows model building with somewhat unfamiliar branes, for which no orientifold planes exist, like NS5-branes.[22] This is particularly interesting, since NS5-branes are a key ingredient in the certain configurations of branes leading to chiral gauge theories, namely brane boxes [50] and brane diamonds [51]. Moreover, this source of chirality in string theory has not been exploited in model building, due to the difficulties in discussing NS-branes in compact examples. In the present section, our purpose is to illustrate the construction of chiral 4d gauge sectors of NS- and D-branes wrapped on generalized calibrated submanifolds.

Consider the background $AdS_5 \times S^5$, with N units of 5-form field strength flux through the S^5. Let us parametrize AdS_5 by the coordinates x^μ, $\mu = 0, \ldots, 3$ and r, and parametrize S^5 as the unit sphere in $\mathbb{R}^6$ parametrized by x^m, $m = 4, \ldots, 9$. The complete metric of the background is

$$ds^2 = H(r)^{-1/2}\, \eta_{\mu\nu}\, dx^\mu\, dx^\nu \,+\, H(r)^{1/2}\,[\,(dx^4)^2 + \cdots + (dx^9)^2\,] \tag{6.1}$$

as discussed in section 3.

We would like to consider a configuration of wrapped calibrated branes in this geometry. Consider a NS5-brane spanning 012, the radial direction r, and the maximal S^2 at $x^7 = x^8 = x^9 = 0$ (hence described as the unit S^2 in the $\mathbb{R}^3$ parametrized by x^4, x^5, x^6). The world-volume is embedded in an $AdS_4 \times S^2$ geometry inside $AdS_5 \times S^5$. Consider a second NS5-brane (denoted NS'-brane) spanning 012, the radial direction r, and the maximal S^2 at $x^5 = x^6 = x^7 = 0$ (hence described as the unit S^2 in the $\mathbb{R}^3$ parametrized by x^4, x^8, x^9). The world-volume is embedded in an $AdS_4 \times S^2$ geometry inside $AdS_5 \times S^5$. These two S^2's intersect at two points, corresponding to $x^4 = \pm 1$ in the S^5.

Finally, introduce a set of n D5-branes spanning 012, the radial direction r, and the maximal S^2 at $x^6 = x^7 = x^9 = 0$ (hence described as the unit S^2 in the $\mathbb{R}^3$ parametrized by x^4, x^5, x^8). The world-volume is embedded in a $AdS_4 \times S^2$ geometry inside $AdS_5 \times S^5$. The complete set of branes and their orientation is sketched in the table 1 where $-$ denotes a direction spanned by the brane volume, and x denotes a direction transverse to it. Also r should be regarded as the radial direction in the $\mathbb{R}^6$ parametrized by x^m.

[22] The analogs of orientifold planes for NS5-branes can be constructed via duality (e.g. as S-dual of O5-planes in type IIB theory). However, this description is not explicit enough to be practical in model building.

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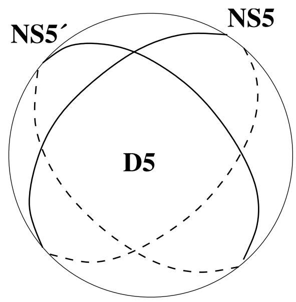

Figure 1: The D5-branes wrap an S^2 in S^5, which is cut in four quadrants by its intersection with the NS- and NS'-brane S^2's.

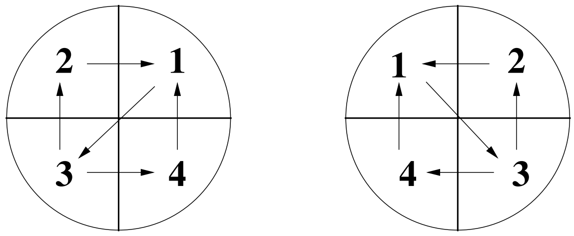

Figure 2: North and South pole views of the S^2 spanned by the D5-branes. The numbers label the gauge groups arising from D5-branes in each quadrant, while the arrows denote chiral multiplets arising from the brane box intersections. Notice that the gluing of the two half-spheres makes some of the arrows in the picture redundant.

All branes are supersymmetric, since they correspond to calibrations of the kind considered in section 3.2.2. Notice that all branes span a common AdS_4, and that in the S^5, the geometry is as follows. Each NS5-brane intersects with the D5-brane in a (different) circle S^1 in S^5. The D5-brane S^2 is therefore cut in four quadrants by these two S^1's, like two orthogonal meridians, which moreover touch at the two poles $x^4 = \pm 1$, as shown in figure 1.

Moreover, it is possible to verify that the configuration preserves 1/8 of the supersymmetries of the background, namely four supercharges, or 4d $\mathcal{N} = 1$ in the common AdS_4 dimensions. Moreover, notice that although we have a compactification on S^5 and a non-trivial set of branes which are stable and supersymmetric, there is no need to introduce orientifold planes, since all branes wrap homologically trivial cycles, and therefore carry no charge.

Interestingly, this set of branes leads to a non-trivial chiral 4d gauge theory in the common AdS_4. One simple way to notice it it to realize that the local geometry near the intersections of the three branes is exactly that arising in brane box models [50], where D5-branes are suspended among intersecting NS-branes. Indeed, the complete configuration can be regarded as two sets of intersecting NS-branes, with D5-branes suspended between them. Since chirality arises from the local intersections between branes, one may use the local features of brane box models to obtain the gauge group and matter content on the chiral gauge theory on the D5-branes. Since the D5-brane S^2 is cut in four pieces by the NS5-branes, the gauge group is $\mathrm{U}(n)^4$. The chiral multiplet content can be described in

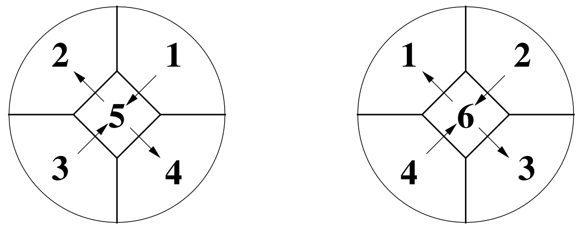

Figure 3: North and South pole view of the calibrated branes in the brane diamond configuration.

terms of arrows stretching between adjacent branes, in a particular way [50]. The set of arrows for our configuration is shown in figure 2, and leads to the chiral multiplet content

$U(n)_1$	$U(n)_2$	$U(n)_3$	$U(n)_4$
$\square$		$\overline{\square}$	
$\square$		$\overline{\square}$	
$\overline{\square}$	$\square$		
$\overline{\square}$			$\square$
	$\overline{\square}$	$\square$	
		$\square$	$\overline{\square}$

An important difference with respect to [50] is that in our model the volumes of the NS-branes are compact, hence they lead to dynamical degrees of freedom in AdS_4. It is difficult to determine them in detail, but they clearly lead to matter uncharged under the D5-brane gauge group, and will not be further discussed. We simply point out that, due to compactness of the NS-branes, the U(1) gauge factors on the D5-branes presumably remain massless, in contrast with standard brane box models.

A more precise version of the brane box models was introduced in [51], where each intersection between NS-branes is resolved into a smooth recombination, leading to a diamond-shaped region, where additional D5-branes can be suspended. In our present setup, the brane diamond configuration is easily obtained by considering our NS-branes to span the holomorphic 2-cycle Σ defined by $z_1 z_2 = \epsilon$, with z_1, z_2 complex coordinates in the 45 and 89 two-planes respectively. An NS5-brane spanning $AdS_4 \times \Sigma$ is calibrated, in analogy with section 3.2.3. The resulting brane diamond picture is shown in figure 3, along with the corresponding arrows, from [51]. The final 4d chiral gauge theory in AdS_4 is

$$\begin{aligned} &\mathrm{U}(n)_1 \times \mathrm{U}(n)_2 \times \mathrm{U}(n)_3 \times \mathrm{U}(n)_4 \times \mathrm{U}(n)_5 \times \mathrm{U}(n)_6 \\ &(\square_1, \overline{\square}_5) + (\overline{\square}_1, \square_6) + (\square_3, \overline{\square}_5) + (\overline{\square}_3, \square_6) + \\ &+ (\overline{\square}_2, \square_5) + (\square_2, \overline{\square}_6) + (\overline{\square}_4, \square_5) + (\square_4, \overline{\square}_6)\,. \end{aligned} \tag{6.2}$$

Clearly, generalizations of the above setup are possible. Introducing a more general network of intersecting NS-branes (or alternatively NS5-branes in more general holomorphic 2-cycles, which are similarly calibrated and supersymmetric) leads to a larger and richer class of 4d chiral gauge theories which can be engineered in this way. It would be interesting to explore the possible phenomenological applications of this kind of construction.

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In this respect it is interesting to point out the possibility of somewhat relaxing the condition to preserve a common supersymmetry, and allow cycles which are calibrated with different phases. Intuitively, this implies that the angles between NS-branes at their intersections do not satisfy the SU(2) condition $\theta_1 \pm \theta_2 = 0$. In compactifications to flat space, such configurations would lead to instabilities against decay to a recombined branes, whose volume is smaller than the intersecting set. In compactifications to AdS space, the potential unstable mode is still above the BF bound, and hence no instability is found, at least for certain range of non-supersymmetric angles.[23]

Another interesting further issue is the possibility of obtaining theories with 4d gravity in the present setup. A simple possibility would be to compactify the direction x^3 of AdS_5, by making it periodic. Unfortunately, its prefactor in the metric makes the circle non-compact at infinity, rendering the gravitational interaction five-dimensional at long distances. Another possibility would be to recover 4d gravity from gravity localization, as in [37], although this does not occur for the Poincare slicing we are exploiting. A less exotic possibility would be embedding the above setups in a global compactification, with a local region of the form $AdS_5 \times S^5$, for instance a la Verlinde [52], or in the way the local Klebanov-Strassler throat [53] is embedded in global compactifications [5]. Since our gauge theory engineering is local, it is valid in any such setup. We leave their detailed discussion for future research.

7. Final comments

In this paper we have described generalized calibrations, and provided examples extending previous generalized calibrated submanifolds, in diverse backgrounds. We have applied this tool to the understanding of supersymmetric wrapped branes in flux compactifications and compactifications on SU(3)-structure manifolds, with several explicit examples, illustrating diverse phenomena. For instance, the use of generalized calibrations allows a neat understanding of the supersymmetry of branes which carry no charges (or at least no $\mathbb{Z}$-valued charges). Also, we have exploited generalized calibrations to show that certain supersymmetric fluxes stabilize D7-brane geometric moduli, via a flux-induced superpotential. This effect is important in the generation of non-perturbative superpotentials from strong dynamics of the D7-brane gauge theory, since the flux modifies the infrared matter content of the theory. Hence, it is crucial in the recently proposed mechanisms to stabilize Kahler moduli in flux compactifications [23] (see [17, 49] for related discussions). In addition, it is related to the interesting question of moduli spaces of generalized calibrated D-branes.

We have also provided a simple characterization of generalized calibrated submanifolds, in terms of the 2- and 3-forms J and Ω present in any SU(3)-structure compactification. We have verified the proposal in several examples of flux compactifications and compactifica-

[23]This follow from the analogous flavoured AdS/CFT analysis in [38], where the inter-brane mode between SU(2) rotated branes corresponded to a tachyon finitely above the Breitenlohner-Freedman bound. Hence, for small enough AdS radius, small deviations from the SU(2) condition do not lead to instabilities.

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tions on half-flat manifolds. It would be interesting to find supporting evidence beyond this class. Presumably, the more complete understanding of the whole class of SU(3)-structure manifolds in [9] will lead to progress in this direction.

Finally, we have suggested that generalized calibrations can also be interesting beyond the above project. In particular, we have offered a new class of compactifications with chiral gauge sectors, based on generalized calibrated brane box configurations. This illustrates the new model building possibilities allowed by generalized calibrations.

We hope much progress in turning generalized calibrations into a central tool in the understanding of string theory in general supersymmetric backgrounds.

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A. Conventions and useful formulae

For carrying out T-duality transformations in the presence of NSNS 3-form flux, it is useful to introduce the notation [7]

$$\begin{aligned} [F_{n(x)}] &= [F_n]_{xi_1\dots i_{n-1}}\, dx^{i_1}\dots dx^{i_{n-1}} \\ g_{(x)} &= \frac{g_{x\alpha}}{g_{xx}} dx^\alpha \\ \omega_{(x)} &= -dg_{(x)}\,. \end{aligned} \tag{A.1}$$

The rules for T-dualization of NSNS and RR fields are taken from [31], see also [54]. They are:

- For NSNS fields:

$$\begin{aligned} \tilde{G}_{xx} &= \frac{1}{G_{xx}} \\ \tilde{G}_{\alpha\beta} &= G_{\alpha\beta} - \frac{1}{G_{xx}}\left(G_{x\alpha}G_{x\beta} - B_{x\alpha}B_{x\beta}\right) \\ \tilde{G}_{x\alpha} &= -\frac{B_{x\alpha}}{G_{xx}} \\ \tilde{B}_{\alpha\beta} &= B_{\alpha\beta} - \frac{1}{G_{xx}}\left(G_{x\alpha}B_{x\beta} - B_{x\alpha}G_{x\beta}\right) \\ \tilde{B}_{x\alpha} &= -\frac{G_{x\alpha}}{G_{xx}} \\ e^{\tilde{\phi}} &= \frac{e^\phi}{\sqrt{G_{xx}}}\,, \end{aligned} \tag{A.2}$$

where we call x the T-duality coordinate. These expressions are valid for going from IIB to IIA or viceversa, and this is the reason why we have not kept the same notation as in the main text for distinguishing IIB and IIA NSNS fields. We distinguish fields in one side and in the other by a tilde. The same applies for RR potentials.

- For RR gauge potentials:

$$\begin{aligned}\tilde{C}^{(n)}_{\alpha\cdots\beta\gamma x} &= C^{(n-1)}_{\alpha\cdots\beta\gamma} - C^{(n-1)}_{[\alpha\cdots\beta|x}\frac{G_{|\gamma]x}}{G_{xx}} \\ \tilde{C}^{(n)}_{\alpha\cdots\beta\gamma\delta} &= C^{(n+1)}_{\alpha\cdots\beta\gamma\delta x} + C^{(n-1)}_{[\alpha\cdots\beta\gamma}B_{\delta]x} + C^{(n-1)}_{[\alpha\cdots\beta|x}\frac{B_{|\gamma|x}G_{|\delta]x}}{G_{xx}}\end{aligned} \tag{A.3}$$

We use a *mostly plus* metric, and the ordinary expression for Hodge dualization in a d-dimensional space, namely:

$$*(dx^{\alpha_1}\wedge\cdots\wedge dx^{\alpha_n}) = \frac{1}{p!}\sqrt{-\det G}\,(g^{\alpha_1\beta_1})\cdots(g^{\alpha_n\beta_n})\epsilon_{\beta_1\cdots\beta_n\beta_{n+1}\cdots\beta_d}dx^{\beta_{n+1}}\wedge\cdots\wedge dx^{\beta_d}\,. \tag{A.4}$$

The convention for the Levi-Civita tensor (with real indices) is $\epsilon_{0123x^1y^1x^2y^2x^3y^3} = +1$.

We use the generalized field strength in the IIB supergravity theory. For those we follow the conventions in [55] (with a shift $\mathcal{B} \longrightarrow -\mathcal{B}$), namely:

$$\tilde{F}_{(2n+1)} = dC_{2n} + \mathcal{H}_3 \wedge C_{(2n-2)} \tag{A.5}$$

which fulfill $\tilde{F}_5 = *\tilde{F}_5$, $\tilde{F}_7 = *\tilde{F}_3$ and $\tilde{F}_9 = *\tilde{F}_1$. Taking into account that in our backgrounds $C_0 = 0$, we end up with the following equations for the dual fields C_6 and C_8:

$$\begin{aligned}dC_6 &= -\mathcal{H}_3\wedge C_4 + *F_3 \\ dC_8 &= -\mathcal{H}_3\wedge C_6\end{aligned} \tag{A.6}$$

with $F_3 = \tilde{F}_3 = dC_2$ in the absence of axion. The expression for the dual gauge potentials is to be obtained from these two equations.

Finally, as already pointed out in the main text, concerning the orientation for the internal holomorphic 4-cycles wrapped by the probe D7 branes in sections 5.3 and 5.4, we consider the volume form $dz^1\wedge d\bar{z}^1\wedge dz^2\wedge d\bar{z}^2$ with positive orientation. As a result, the volume form used in our calibrations $dx^1\wedge dy^1\wedge dx^2\wedge dy^2$ has negative orientation.

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